

Let  $P$  be an abstract 3-dimensional polytope.

We say that a face  $f_i$  of  $P$  is *adjacent* to a face  $f_j$  if  $f_i \cap f_j$  is an edge.

By a *prismatic  $k$ -circuit* we mean  $k$  facets  $f_1, \dots, f_k$  of  $P$  such that

- $f_i$  has a common side with  $f_{i+1}$  for all  $i = 1, \dots, k$ , where  $f_{k+1} := f_1$ ;
- no three of  $f_1, \dots, f_k$  have a common vertex.

**Notation.** if  $f_i$  is adjacent to  $f_j$ , denote by  $\alpha_{ij}$  the angle between  $f_i$  and  $f_j$ .

**Theorem [And1].**

A compact acute-angled polytope  $P$  of a given simple combinatorial type with given dihedral angles exists in  $\mathbb{H}^3$  if and only if the following conditions hold:

- (1) if three faces  $f_i, f_j, f_k$  pass through the same vertex then  $\alpha_{ij} + \alpha_{jk} + \alpha_{ki} > \pi$ ;
- (2) if the faces  $f_i, f_j, f_k$  form a 3-circuit, then  $\alpha_{ij} + \alpha_{jk} + \alpha_{ki} < \pi$ ;
- (3) if four faces  $f_i, f_j, f_k, f_l$  form a 4-circuit then  $\alpha_{ij} + \alpha_{jk} + \alpha_{kl} + \alpha_{li} < 2\pi$ ;
- (4) if  $P$  is a *triangular prism* then at least one dihedral angle between a triangular side and a quadrilateral side is not equal to  $\frac{\pi}{2}$ ;
- (5) if  $P$  is a *tetrahedron* then the determinant of the Gram matrix should be negative.

**Theorem [And2].**

A finite volume acute-angled polytope  $P$  of a given simple in edges combinatorial type with given dihedral angles exists in  $\mathbb{H}^3$  if and only if the following conditions hold:

- (1a) if three faces  $f_i, f_j, f_k$  pass through the same vertex then  $\alpha_{ij} + \alpha_{jk} + \alpha_{ki} \geq \pi$ ;
- (1b) if four faces pass through the same vertex then all angles formed by them are equal to  $\frac{\pi}{2}$ ;
- (2a) if the faces  $f_i, f_j, f_k$  form a 3-circuit, then  $\alpha_{ij} + \alpha_{jk} + \alpha_{ki} < \pi$ ;
- (2b) if  $f_i$  is adjacent to  $f_j$  and  $f_k$ , and  $f_j$  is not adjacent to  $f_k$ , but  $f_j \cap f_k$  is an ideal vertex, then  $\alpha_{ij} + \alpha_{ik} < \pi$ ;
- (3) if four faces  $f_i, f_j, f_k, f_l$  form a 4-circuit then  $\alpha_{ij} + \alpha_{jk} + \alpha_{kl} + \alpha_{li} < 2\pi$ ;
- (4) if  $P$  is a *triangular prism* then at least one dihedral angle between a triangular side and a quadrilateral side is not equal to  $\frac{\pi}{2}$ ;
- (5) if  $P$  is a *tetrahedron* then the determinant of the Gram matrix should be negative.